Gravitation

(gravitational PE, escape velocity, orbital velocity and geostationary satellite)

Note : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

Gravitational potential energy

Gravitational potential of a system of two masses is the work done to assemble these mass from infinity.

$$\boxed{U = -\frac{Gm_1 m_2}{r}}$$

Gravitational potential energy of earth-body system

Gravitational potential of a body and the earth when their centre of masses are at a distance of R is

$$\boxed{U = -\frac{GMm}{R}}$$

Gravitational potential energy of earth-body system

Gravitational force acting on the body at this point is

$$F = \frac{GMm}{r^2}$$

Work done (dW) is causing a small displacement is

$$\mathrm{d}W = \bar{F} \bullet \mathrm{d}\bar{r}$$

$$\mathrm{d}W = F\ \mathrm{d}r$$

$$\mathrm{d}W = \frac{GMm}{r^2}\ \mathrm{d}r$$

Total work done from infinity to a distance R from centre of the earth is

$$W = \int_{\infty}^{R} \frac{GMm}{r^2}\ \mathrm{d}r$$

$$W = GMm\int_{\infty}^{R} \frac{1}{r^2}\ \mathrm{d}r$$

$$W = -GMm\left[\frac{1}{R} - \frac{1}{\infty}\right]$$

$$W = -\frac{GMm}{R}$$

This work is stored as potential energy, therefore

$$\boxed{U = -\frac{GMm}{R}}$$

Escape speed (v_e)

The minimum speed with which a body should be projected from the earth so that it eventually escapes the earth's gravitational pull.

Total initial energy of the system when projected from the surface of the earth is

$$TE = KE + PE$$

$$TE = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

Total final energy of the system when the body escapes the earth's pull is

$$TE = KE + PE$$

$$TE = 0 + 0$$

Using law of conservation of energy

$$TE_{\text{initial}} = TE_{\text{final}}$$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$v^2 = \frac{2GM}{R}$$

$$\boxed{v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}}$$

Orbital speed (v_e)

Speed required for a body to revolve around the earth in a stationary orbit.

Gravitational force acting on a body of mass m orbiting the earth at a height h from its surface is

$$F = \frac{GMm}{(R+h)^2}$$

Force required to maintain a circular path (centripetal force) is

$$F = \frac{mv^2}{(R+h)}$$

Gravitational force provides the required centripetal force, therefore

$$\frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$$

$$v^2 = \frac{GM}{(R+h)}$$

$$\boxed{v_o = \sqrt{\frac{GM}{(R+h)}}}$$

For low altitudes of orbits ($h << R$) we get

$$\boxed{v_o \approx \sqrt{gR}}$$

Geostationary satellite

A satellite that appears to be stationary for an observer located on the earth.

Conditions for a satellite to be geostationary

Its direction of revolution and time period of revolution should be same as that of the earth

Uses of a geostationary satellite

1. Weather forecasting
2. Communication